Modern Theories of Evidence and the Concept of Belief in Islamic Perspective

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1. Introduction

Human mind is a residence of different concepts, ideas, conjectures, hypotheses, information etc., and if these residents attract the attention of someone's mind, it becomes his belief. So belief can be considered as the inclination of someone's mind towards truth or untruth of these residents or it can be considered as the acceptance of someone's mind about truth or untruth of the residents. It is a state of human mind that depends on evidence. More evidence someone acquires in favour of a resident of his mind, more will be his strength of belief in the resident.

Belief in Allah is the first and foremost principle in Islam and the Holy Qur´ān cites various signs (evidences), for those who understand:

Most surely in this, there are signs for people who understand

(Ar-rād:4)

Surely in the making of the heavens and earth, And the alternation of the night and the day, There are signs for people of understanding

(Āl-e-Imrān:190)

and the consequence of this belief are various other beliefs:

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O' ye who believe!
Believe in God,
And His Messenger,
And the Book, which He has caused to be sent down upon His Messenger,
And the Books, which He has sent down before,
And he, who chooses to disbelieve in God,
And His Messenger spirits,
And His Books,
And His Messengers,
And the Future Day,
Has than indeed, lost by a long-losing (Way)
(An-Nisā:136)

2. Quantification of Belief

Belief is subjective i.e. different persons with the same set of evidences acquire different degrees of belief. Although it is not easy to quantify belief, but in many situations, like if someone is going to computerize his knowledge about belief, it is necessary to do so. The Bayesian theory of estimation and the Dempster-Shafer theory of evidence are some of the approaches to estimate it. Even if a person's degree of belief is subjective, there are many situations where, in addition to subjective approach, it can be objectively treated alike probability of an event that may be interpreted by subjective as well as objective approaches, e.g. in a situation, like throwing a pair of dice, the belief can be supported objectively while in other situations, such as predicting the price of gold for future, any probability used is an essentially subjective.

"The Classical or A priori Definition of Probability", which is an objective approach says:

If a random experiment can produce n mutually exclusive and equally likely outcomes and if m out of these outcomes are considered favourable to the occurrence of a certain event A, then the probability of the event A is defined as the ratio $\frac{m}{n}$.

"The Relative Frequency or A Posteriori Definition of Probability", which is also an objective approach says:

If a random experiment is repeated as a large number of times, say n, under identical conditions and if an event A is observed to occur m times, then the probability of the event A is defined as the limit of the relative frequency $\frac{m}{n}$ *as n tends to infinity.* Symbolically, we write

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

According to the "Subjective or Personalistic Probability" approach, probability may be defines as a measure of the degree of confidence or belief that a particular individual has in the occurrence of an event A. Probability in this sense is purely subjective and is based on whatever evidence is available to the individual. The subjective probability has a disadvantage that two or more persons faced with the same evidence may arrive at different probabilities.

The Bayesian approach of estimation is a subjective approach. This approach combines sample information with other available prior information that may appear to be pertinent. These information(evidences) estimate subjective probabilities which are numerical measures of epistemic belief. A person uses his own experience and knowledge as the basis for arriving at these probabilities.

Let events H and $\overline{H}$ form a partition of a sample space S i.e. H and $\overline{H}$ are mutually exclusive events and $H \cup \overline{H} = S$

then Baye's theorem is expressed as

$$P(\frac{H}{E}) = \frac{P(H)P(\frac{E}{H})}{P(E)}$$

where, $P(E) = P(H) . P(\frac{E}{H}) + P(\overline{H}) . P(\frac{E}{\overline{H}})$

The interpretation of various factors in Baye's theorem may be given as follows:

$P(H)$: Prior probability of a hypothesis.

$P(\frac{E}{H})$: Probability of evidence, given the hypothesis.

$P(E)$: Probability of evidence.

$P(\frac{E}{\overline{H}})$: Probability of evidence, given the negation of the hypothesis.

$P(\frac{H}{E})$: Posterior probability of hypothesis given the evidence.

The prior probability of negation of the hypothesis H is:

$$P(\overline{H})=1-P(H)$$

and the posterior probability for the negation of the hypothesis, using Baye's rule, can be obtained as

$$P(\frac{\overline{H}}{E})=\frac{P(\overline{H}).P(\frac{E}{\overline{H}})}{P(E)}$$

Let's now examine a problem where Baye's rule can be used most appropriately to determine the posterior probability associated with the hypothesis, if the evidence is given.

Suppose an urn contains four balls, which are known to be either (i) all white or (ii) two white and two black. A ball is drawn at random and is found to be white. Here, we have two hypotheses H and $\overline{H}$ and an evidence E, where

H: all the balls are white.
$\overline{H}$: two balls are white and two are black (i.e. all the balls are not white)
E: a ball drawn is found to be white.

Now before the ball is drawn, the probabilities of H and $\overline{H}$ i.e. P (H) = P ($\overline{H}$) = $\frac{1}{2}$, which are prior probabilities and are the initial degrees of belief, before receiving any evidence.

since $P(\frac{E}{H})= 1$ and

$$P(\frac{E}{\overline{H}})=\frac{1}{2}$$

so $P(E) = P(H).P(\frac{E}{H}) + P(\overline{H}).P(\frac{E}{\overline{H}})$

$= \frac{1}{2}.1 + \frac{1}{2}.\frac{1}{2} = \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4}$

or $P(E) = \frac{3}{4}$

Therefore, the posterior probabilities are:

$$P(\frac{H}{E}) = \frac{P(H).P(\frac{E}{H})}{P(E)} = \frac{\frac{1}{2}.1}{\frac{3}{4}} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3} \quad \textit{and}$$

$$P(\frac{\overline{H}}{E}) = \frac{P(\overline{H}).P(\frac{E}{\overline{H}})}{P(E)} = \frac{\frac{1}{2}.\frac{1}{2}}{\frac{3}{4}} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$

Which are the degrees of belief after receiving evidence. So the person who draws the ball and arrives at these probabilities, prefers the first hypothesis i.e. all the balls are white as it has larger posterior probability.

Now suppose, the person places the ball back into the urn and another person draws a ball, which is black. So here

E: ball drawn is black.

$P(\frac{E}{H}) = 0$

$P(\frac{E}{\overline{H}}) = \frac{1}{2}$

$P(E) = P(H).P(\frac{E}{H}) + P(\overline{H}).P(\frac{E}{\overline{H}})$

$$= \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Therefore, the posterior probabilities

$$P\left(\frac{H}{E}\right) = \frac{P(H).P\left(\frac{E}{H}\right)}{P(E)} = \frac{\frac{1}{2}.0}{\frac{1}{4}} = 0 \quad \textit{and}$$

$$P\left(\frac{\overline{H}}{E}\right) = \frac{P(\overline{H}).P\left(\frac{E}{\overline{H}}\right)}{P(E)} = \frac{\frac{1}{2}.\frac{1}{2}}{\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$$

Which are the degrees of belief of the second person after receiving evidence. The second person, therefore, totally rejects the first hypothesis and accepts the second one with 100 percent certainty, i.e. he believes that the urn contains two white and two black balls.

3. Influence of Evidence on Belief

Posterior probability of a hypothesis depends on evidence. The prior probability of the hypothesis H in the "black and white balls" problem (section 2) increases for the first person with the evidence that the ball drawn is white while it decreases for the second person with the evidence that the ball drawn is black. In general:

1) posterior probability of a hypothesis < prior probability of the hypothesis or
2) posterior probability of a hypothesis = prior probability of the hypothesis or

3) posterior probability of a hypothesis > prior probability of the hypothesis

A measure that explains the degree to which the observation of evidence E influences the prior probability of hypothesis H is known as *level of sufficiency* (LS). It is a ratio (likelihood ratio) of the probability of evidence, given the hypothesis H to the probability of evidence, given the complement of the hypothesis. i.e.

$$LS = \frac{P(\frac{E}{H})}{P(\frac{E}{\bar{H}})}$$

There is another measure, *level of necessity* (LN), which represents the degree to which the observation of complement of the evidence E i.e. Ē influences the prior probability of the hypothesis H. It is a ratio of the probability of Ē, given the hypothesis H to the probability of Ē, given the complement of the hypothesis. i.e.

$$LN = \frac{P(\frac{\bar{E}}{H})}{P(\frac{\bar{E}}{\bar{H}})}$$

The values of both the *level of sufficiency* and the *level of necessity* lie in the interval $(0,\infty)$ and they may be divided into three categories i.e. less than, equal to or greater than 1. Now:

1) if LS < 1 then LN > 1 and the posterior probability < prior probability (i.e. LS < 1 indicates that the

observation of E tends to confirm the negation of the hypothesis H which is $\overline{H}$)

2) if LS = 1 then LN = 1 and the posterior probability = prior probability
3) if LS > 1 then LN < 1 and the posterior probability > prior probability (i.e. LS > 1 indicates that the observation of E tends to confirm the hypothesis H)

Let's reshape the " black and white balls" problem in the form

H: m white and n black balls.

$\overline{H}$: p white and q black balls.

E: a ball drawn is white.

Ē: ball drawn is not a white ball (i.e. it is black).

Then

$$P(H)=\frac{1}{2}$$

$$P(\overline{H})=\frac{1}{2}$$

$$P(\frac{E}{H})=\frac{m}{m+n}$$

$$P(\frac{E}{\overline{H}})=\frac{p}{p+q}$$

$$P(\frac{\overline{E}}{H})=\frac{n}{m+n}$$

$$P(\frac{\overline{E}}{\overline{H}})=\frac{q}{p+q}$$ and therefore

(1) $$P(\frac{H}{E})=\frac{m(p+q)}{2mp+mq+np}$$

(2) $P(\frac{\overline{H}}{\overline{E}}) = \frac{q(m+n)}{2nq+mq+np}$

(3) $LS = \frac{m(p+q)}{p(m+n)}$ and

(4) $LN = \frac{n(p+q)}{q(m+n)}$

The figure1 is formed with $m + n = p + q = 4$.It depicts how E and Ē influence the prior probabilities P(H) and $P(\overline{H})$ respectively

The two hypotheses H and $\overline{H}$ are identical for the rows 1,7,13,19, and 25 where $m = p$ and $n = q$, i.e. there exists a single hypothesis in those cases. Moreover, for the row 1, $P(\frac{H}{E})$ is undefined as in this case the hypothesis H is "the urn contains 0 white and 4 black balls"

and the evidence E is "the ball drawn is white" which is inconsistent. Similarly, for the row 25, $P(\frac{H}{\overline{E}})$ is undefined as in this case there are 4 white and 0 black balls in the urn and the

ball drawn is black which is also inconsistent. For the remaining rows of identical hypotheses, i.e. row 7, 13, and 19, the two posterior probabilities should be added together

row #	m	n	p	q	LS	$P(\frac{H}{E})$	LN	$P(\frac{H}{\bar{E}})$
1	0	4	0	4	undefined	undefined	1	0.5
2	1	3	0	4	undefined	1	0.75	0.43
3	2	2	0	4	undefined	1	0.5	0.33
4	3	1	0	4	undefined	1	0.25	0.2
5	4	0	0	4	undefined	1	0	0
6	0	4	1	3	0	0	1.33	0.57
7	1	3	1	3	1	0.5	1	0.5
8	2	2	1	3	2	0.66	0.66	0.4
9	3	1	1	3	3	0.75	0.33	0.25
10	4	0	1	3	4	0.8	0	0
11	0	4	2	2	0	0	2	0.66
12	1	3	2	2	0.5	0.33	1.5	0.6
13	2	2	2	2	1	0.5	1	0.5
14	3	1	2	2	1.5	0.6	0.5	0.33
15	4	0	2	2	2	0.66	0	0
16	0	4	3	1	0	0	4	0.8
17	1	3	3	1	0.33	0.25	3	0.75
18	2	2	3	1	0.66	0.4	2	0.66
19	3	1	3	1	1	0.5	1	0.5
20	4	0	3	1	1.33	0.57	0	0
21	0	4	4	0	0	0	undefined	1
22	1	3	4	0	0.25	0.2	undefined	1
23	2	2	4	0	0.5	0.33	undefined	1
24	3	1	4	0	0.75	0.43	undefined	1
25	4	0	4	0	1	0.5	undefined	undefined

Figure 1: Influence of E and $\bar{E}$ on the prior probabilities $P(H)$ and $P(\overline{H})$ respectively.

to get $P(\frac{H}{E}) = P(\frac{H}{\overline{E}}) = 0.5 + 0.5 = 1$. The value of $P(\frac{H}{E})$ in rows 2, 3, 4, and 5 is 1, where as $P(\frac{\overline{H}}{E}) = 1 - P(\frac{H}{E}) = 0$, indicating that the evidence of receiving a white ball, totally rejects the second hypothesis (the urn contains 0 white and 4 black balls) and accepts the first one (there is at least one white ball) with 100% certainty.

4. Categories of Belief

Allah says in the Holy Qur´ān:

"O ye who believe! Avoid presumption in most cases, for surely presumption in some cases is a sin"

(Al-Hūjūrāt: 12)

The word *presumption* is the translation of the Arabic word *zann*. It is first to know what is zann, together with its vicinal.

Whenever a concept, information, a hypothesis, an idea etc. appears into someone's mind, the mind either inclines towards its truth or untruth, or it does not.

1) If the mind does not incline towards its truth or untruth then it is *takhay´yūl* (imagination). (see figure 2)
2) If the mind inclines towards either side i.e. either towards its truth or towards its untruth and the inclination is:

- 2.1) against the fact and invariant then it is *yaqīn-e-kāzib* or *jehl-e-murak'kab* (gross ignorance).
- 2.2) in favour of or against the fact but variant then it is *taqlīd* (conformity)
- 2.3) in favour of the fact and invariant then it is *yaqīn-e-sādiq* (sound assurance).

3) If the mind inclines both towards its truth and untruth and the inclination is
   - 3.1) same towards both sides then it is *shak* (doubt).
   - 3.2) different for each side then the side where it is
     - 3.2.1) lesser is the side of *wahm* (hallucination).
     - 3.2.2) greater is the side of *zann* (presumption).

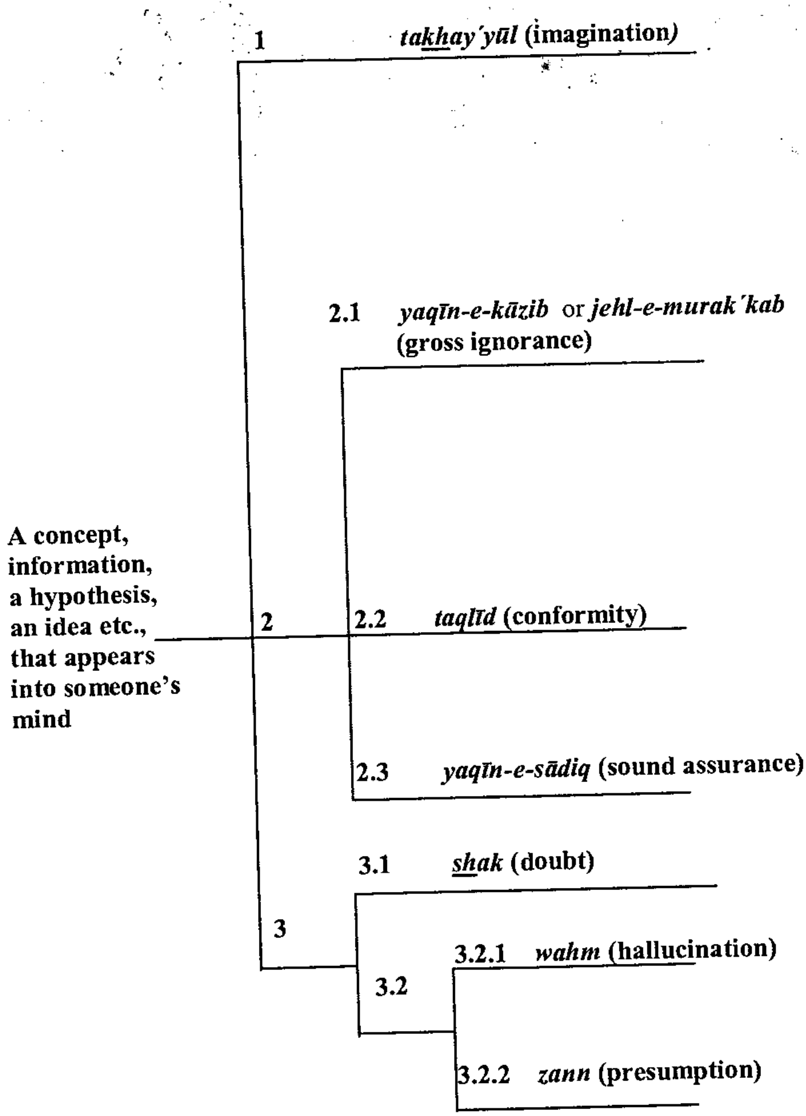

Figure 2: Different categories of belief.

The terms, which are shown in the tree diagram, can be described in another way as follow:

1) If degree of belief (for each of the truth or untruth) = 0% then the idea or concept is *takhay'yūl*. (see figure 3)
2) If 0% < degree of belief (for either) < 50%, then it is *wahm*.

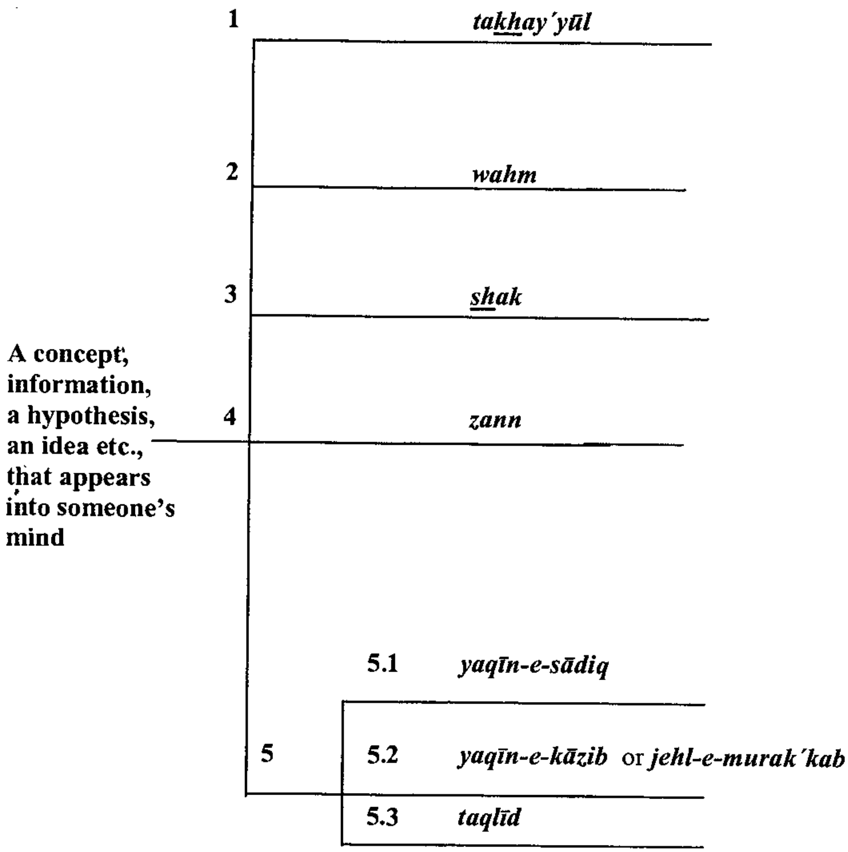

Figure 3: Different categories of belief (another approach).

3) If degree of belief (for each) = 50%, then it is *shak*.
4) If 50% < degree of belief (for either) < 100%, then it is *zann*.
5) If degree of belief (for either) = 100%, then it is *yaqīn* (assurance).

Moreover:

5.1) *yaqīn* in favour of a fact and invariant is *yaqīn-e-sādiq*,
5.2) *yaqīn* against the fact and invariant is *yaqīn-e-kāzib* or *jehl-e-urak'kab*, and
5.3) *yaqīn* in favour of or against the fact but variant is *taqlīd*.

Initially it is *wahm* that appears into someone's mind, if an apparent indication is observed. This *wahm*, which is *idtirārī* (out of control), is not a sin. Now this *wahm* is either necessary or unnecessary *wahm*. A *wahm*, which is required to explore whether it is true or not, is necessary e.g., parents of someone find certain apparent indications in the behaviour of their child and feel that something is going wrong or a police officer is going to investigate, when he observes certain indications that something is going wrong, etc. are of this category. Besides these, there are so many others, which belong to the category of unnecessary *wahm*. If the *wahm* is unnecessary, it should be thrown out of mind, other wise it grows up, first converts into *shak* and finally into *zann*. This *zann* is *ikh'tīārī* (under control). It is a sin and is ordered to avoid. The necessary *wahm* when grows up, converts into such kind of *zann* which is not a sin, although it is *ikh'tīārī* too, and is not to be avoided necessarily but for the sake of caution, some of these lawful *zann* are also to be avoided. Note that the Holy Qur'ān says: *"presumption in some cases is a sin"*. It does not say that it is a sin in all cases i.e. it is not *harām* (unlawful) in all cases. It is also to be noted that although presumption in some cases is a sin, but according to the Holy Qur'ān, *"Avoid presumption in most cases"*. This is due to the fact that some cases of presumptions, which are *harām* and are sin, together with some *ghair-harām* (lawful) cases, which are not sin, form the *"most cases"*. Moreover it is *tajassūs* (curiosity) that supports the conversion of a *wahm* into a *zann* and the Holy Qur'ān says" *lā tajassasū*" (don't spy). It is that *tajassūs* which converts unnecessary *wahm* into unnecessary *zann*. In other cases, presumption falls into other categories of deeds i.e., it is either *farḍ* (obligatory), *wājib* (unavoidable), *mūs'tahab* (desirable), *mūbāh* (permissible), *makrooh-e-tanzīhī* (abominable), *makrooh-e-tahrīmī* (unallowable) or *harām* (unlawful).

5. The structure of *Ahkām-e-Sharī'ah*

The *dīn-e-Islam* is basically consists of *'aqā'id* and *a'māl*. *A'māl* are sub divided into *akhlāq, mū'āmalāt, mū'āsharat* . and *'ibādāt*. These *a'māl* are *farḍ, wājib, mūs'tahab, mūbāh, makrooh-e-tanzīhī, makrooh-e-tahrīmī* or *harām* and are categorized according to those which are ordered by Allah or which are prohibited. Although there are $2^2=4$ cases:

1. ordered by Allah
2. neither ordered nor prohibited and is therefore permissible

3. prohibited by Allah
4. ordered by Allah as well as prohibited,

but the fourth case is logically impossible.

Case#1 can further be divided into that, which is deduced or established by:

1.1. *dalīl-e-qat'ī* (definitive evidence)
1.2. *dalīl-e-zannī* (conjectural or presumptive evidence)

The deed, which is according to case#1.1 is *fard* . (obligatory), while the other which is according to case#1.2 is *wājib* (unavoidable). (see figure 4)

Case#2 can further be divided into:

2.1 likeable
2.2 neither likeable nor dislikeable but are neutral
2.3 dislikeable

The deed, which is according to:

case#2.1 is *mūs'tahab* (desirable)
case#2.2 is *mūbāh* (permissible)
case#2.3 is *makrooh-e-tanzīhī* (abominable).

Case#3 can further be divided into that:

3.1. which is deduced or established by *dalīl-e-zannī* only or which is although deduced by *dalīl-e-qat'ī* but is *ghair-muttafaq-un- alaih*.
3.2 which is not only deduced or established by *dalīl-e-qat'ī* but is also *muttafaq-un- alaih*

The deed, which is according to case#3.1, is *makrooh-e-tahrīmī* (unallowable) while the other, which is according to case#3.2, is *harām* (unlawful).

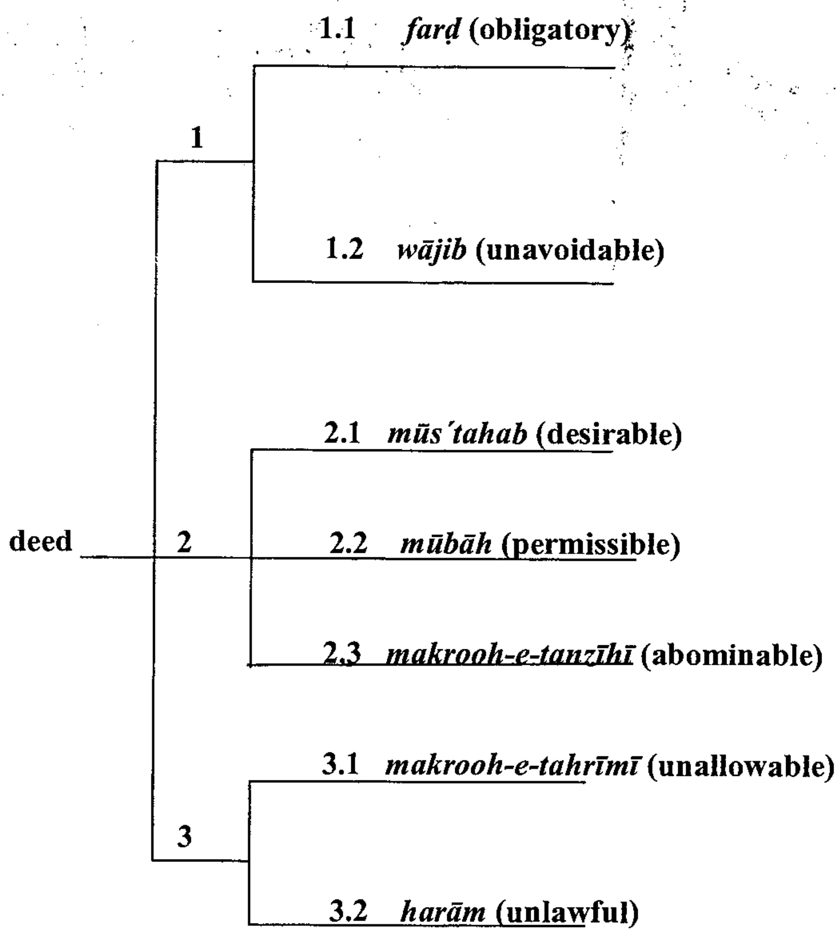

Figure 4: Categories of deed.

6. Dempster-Shafer Theory of Evidence

The use of Baye's rule for manipulation of measures of belief is often regarded as inappropriate. Treating probability as a measure of belief, the law of additivity for a hypothesis and its complement is one of the drawbacks of the Bayesian model. Belief measures should not have to consider exactly same as probabilities, for instance, the sum of probabilities of the hypothesis H and its complement $\overline{H}$ discussed in the "black and white balls" problem (section 2), $P(H) + P(\overline{H}) = 1$, whereas the sum of beliefs, on the other hand, $Bel(H) + Bel(\sim H) \leq 1$. This is so because belief depends on evidence. In the absence of an evidence for a hypothesis H and for its negation ~H, the sum $Bel(H) + Bel(\sim H) = 0$ and in other cases, this sum lies in the interval $(0,1]$

Another drawback of the Bayesian model is that according to it, a hypothesis is assigned a single value, which may be not correct. Furthermore, there is no way to differentiate between ignorance and uncertainty in the Bayesian model.

The Dempster-Shafer theory of evidence provides an alternative, more general, model for the assessment of numerical degree of belief. It can be considered an extension of the Bayesian approach. It is a well-known procedure for reasoning with uncertainty in *Artificial Intelligence**.

* A branch of computer science. It is the structured development of theory, methodology, and physical system that enable computer and robots to perform tasks that historically have been judged to lie within the domain of intelligence.

It distinguishes between uncertainty and ignorance by creating belief functions. Belief functions allow us to use our knowledge to bind the assignment of probabilities when these may be unavailable.

According to figure #4, some acts or deeds fall into the category of *ghair-harām* and the remaining into the category of *harām*. We recognize some of the *ghair-harām* deeds as a *ghair-harām* and some of the *harām* as a *harām*. The remaining of *ghair-harām* and *harām*, for which we haven't any knowledge, fall into gray area i.e. we have ignorance or lack of information for such type. The *ghair-harām* can be sub categorized into *mūs'tahab, mūbāh, makrooh-e-tanzīhī* etc., and there are so many gray areas exist there. The Dempster-Shafer theory can be applied in such areas. The theory can be used to attempt to convert those gray areas either into black or into white.

This approach is especially appropriate for combining expert opinion, since experts do differ in their opinion with a certain degree of ignorance and, in many situations, at least some epistemic information (i.e., one that was constructed from vague perceptions).

The Dempster-Shafer theory provides a model for the assessment of numerical degree of belief. This approach can be used to handle epistemic information as well as ignorance or lack of information. It considers sets of propositions and assigns to each of them an interval

[Belief, Plausibility]

which contains the information about the degree of belief in a set of propositions. Belief measures the strength of evidence in favour of set of propositions. It ranges from 0%(indicating no evidence) to 100% (denoting certainty)

Plausibility is defined to be

Plausibility (H) = 1 – Belief (~H)

It also ranges from 0% to 100% and measures the extent to which evidence in favour of ~H leaves room for belief in H. The belief-plausibility interval [Belief, Plausibility] measures not only our level of belief in some propositions, but also the amount of information we have.

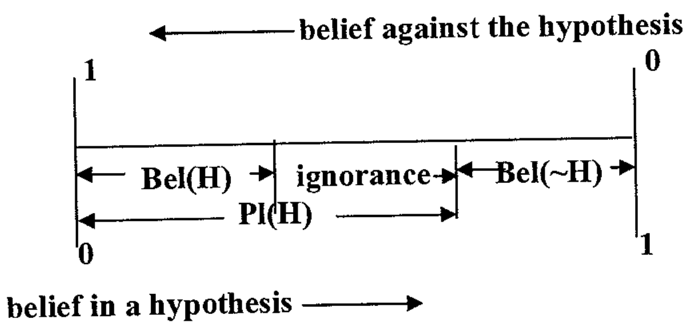

Figure 5: Belief, plausibility and ignorance.

The strategy followed in the Dempster-Shafer theory for dealing with uncertainty roughly amounts to starting with an initial set of hypotheses, then for each piece of evidence associating a measure of uncertainty with certain subsets of the original set of hypotheses until measures of uncertainty may be associated with all possible subsets on account of the combined evidence. The initial set of all hypotheses in the problem domain is called the *frame of discernment*. In such a frame of discernment the individual hypotheses are assumed to be disjoint. The impact of a piece of evidence on the confidence or belief in certain subsets of a given frame of discernment is described by means of a function which is defined below.

Let Θ be a frame of discernment. If with each subset $X \subseteq \Theta$ a number m (X) is associated such that:

(1) $0 \leq m(X) \leq 1$

(2) $m(\varnothing) = 0$

(3) $\sum_{X \subseteq \Theta} m(X) = 1$

then m is called a *basic probability assignment* or simply bpa on Θ.For each subset $X \subseteq \Theta$, the number m(X) is called the *basic probability number* of X.

A probability number m(X) expresses the confidence or belief assigned to precisely the set X; it does not express any belief in subsets of X. It will be evident, however, that the total confidence in X is not only dependent on the confidence in X itself, but also on the confidence assigned to subsets of X.

Although there is a great resemblance between a basic probability assignment and a probability function but in some cases, they differ too. A probability function p associates with each element in Θ a number from the interval [0,1] such that the sum of these numbers equals 1 i.e., p: $\Theta \rightarrow [0,1]$ and $\sum_{x \subseteq \Theta} p(X) = 1$; a basic probability assignment m associates with each element in 2^{Θ} a number in the interval [0,1] such that once more the sum of the numbers equal 1 i.e., m: $2^{\Theta} \rightarrow [0,1]$ and $\sum_{X \subseteq \Theta} m(X) = 1$.

Moreover, basic probability assignment purely is subjective, that depends on evidence.

7. Dempster's rule of combination

The Dempster-Shafer theory is based on two ideas: first, the idea of obtaining degrees of belief for one question from subjective probabilities for related questions, and second, the use of a rule for combining these degrees of belief when they are based on independent items of evidence.

Let's suppose a deed, which is neither *farḍ, wājib* or *makrooh-e-tahrīmī* nor *harām,* is to be investigated whether it is likeable, neither likeable nor dislikeable or it is dislikeable, i.e. whether it is *mūs'tahab, mūbāh,* or *makrooh (makrooh-e-tanzīhī)*

The tree diagram of the frame of discernment, which is a portion of figure 4, is given below:

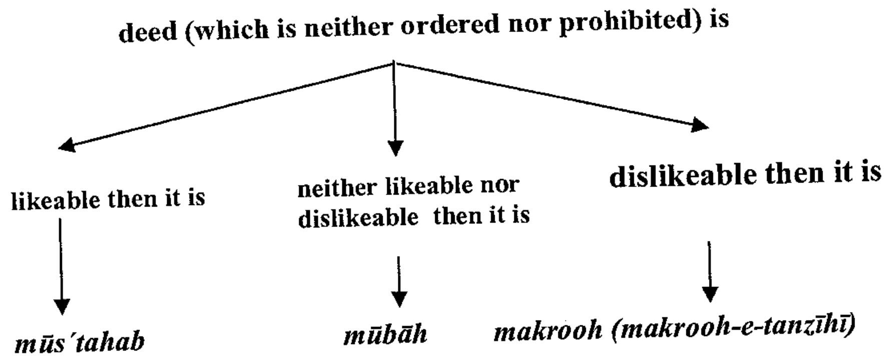

Figure 6: Sub-categories of deed.

Initially, our belief is in the whole set Θ = {*mūs'tahab, mūbāh, makrooh*} which is the frame of discernment. Each bpa on Θ assigns basic probability numbers to $2^4 = 16$ sets (including the empty set). As there is no evidence pointing at a certain kind in particular, the Dempster-Shafer theory assigns the basic probability number 1,which is the strength of our belief, to the entire frame of discernment and each proper subset of the frame of discernment gets assigned the number 0:

$$m_0(x) = \begin{cases} 1 & \text{if } X = \Theta \\ 0 & \text{otherwise} \end{cases}$$

Definition: Let Θ be a frame of discernment and let *m* be a basic probability assignment on Θ.A set $X \subseteq \Theta$ is called a *focal element* in *m* if *m*(X) > 0 i.e., The sets which do have a non-zero bpa are referred to as focal elements. The *core* of *m* is the set of all focal elements in *m or* the set that contains the union of the focal elements is the core of the bpa.

So the core of m_0 is equal to $\{\Theta\}$. Now suppose that some evidence has become available indicating that the deed is not dislikeable i.e., either it is likeable (*mūs'tahab*) or it is neither likeable nor dislikeable (*mūbāh)*; let the strength of our belief is expressed in a number 'a', where $0 \leq a \leq 1$.This 'a' is assigned to the subset *{mūs'tahab, mūbāh}*, and is viewed as the composite hypothesis mūs'tahab or mūbāh. Because of lack of further information, the remaining certainty '1-a' is assigned to the entire frame of discernment; there is no number associated with the individual hypotheses, because more specific information, indicating which of the two kinds of deed is it, is not available.

For a=0.2:

$$m_1(x) = \begin{cases} 0.8 \text{ if } X = \Theta \\ 0.2 \text{ if } X = \{\text{mūs'tahab, mūbāh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

The set *{mūs'tahab, mūbāh}* is an element of the core of m_1. Now suppose that we have furthermore obtained some evidence against the hypothesis that the kind of the deed is mūbāh. This information can be considered as support for the hypothesis that the kind is *not* mūbāh. This latter hypothesis is equivalent to the composite hypothesis mūs'tahab or makrooh. In consequence of this evidence, we therefore assign a basic probability number, for example 0.9, to the set *{mūs'tahab, makrooh}*:

$$m_2(x) = \begin{cases} 0.1 \text{ if } X = \Theta \\ 0.9 \text{ if } X = \{\text{mūs'tahab, makrooh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

The Dempster-Shafer theory provides a function for computing from two pieces of evidence and their associated basic probability assignment a new basic probability assignment describing the combined influence of these pieces of evidence. This Function is known as Dempster's rule of combination.

Definition: {Dempster's rule of combination}: Let Θ be a frame of discernment, and let m_1 and m_2 be basic probability assignments on Θ. Then, $m_1 \oplus m_2$, known as orthogonal sum; is a function

$$m_1 \oplus m_2 : 2^{\Theta} \rightarrow [0,1] \text{ such that:}$$

(1) $m_1 \oplus m_2(X) = 0$, for $X = \varnothing$ and

(2) $m_1 \oplus m_2(X) = \dfrac{\sum_{Y \cap Z = X} m1(Y)\, m2(Z)}{1 - \sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z)}$ for all $X \neq \varnothing$, where

$\sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z)$, present in the denominator, is the 'weight of conflict' and the denominator $1 - \sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z)$ itself is the 'normalization coefficient' or the 'scaling factor'. Now:

(a) If there is no conflict between the two evidences then $\sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z)$ is zero and the normalization coefficient will become 1

(b) If there is a partial conflict between the two evidences then $0 < \sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z) < 1$ and therefore $0 < 1 - \sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z) < 1$ (note in both of the cases (a) and (b), the weight of conflict is less than unity and therefore for each of the cases, the orthogonal sum is well defined)

(c) If there is a total conflict between the two evidences then $\sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z) = 1$ and the normalization coefficient will become zero, leaving the combination of contradictory evidence undefined (m_1 and m_2 in such case is 'flatly contradictory').

The two results of basic probability assignments obtained from the two different evidences are summarized below:

$$m_1(x) = \begin{cases} 0.8 \text{ if } X = \Theta \\ 0.2 \text{ if } X = \{\text{mūs'tahab, mūbāh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

$$m_2(x) = \begin{cases} 0.1 \text{ if } X = \Theta \\ 0.9 \text{ if } X = \{\text{mūs'tahab, makrooh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

From applying Dempster's rule of combination, we obtain a new basic probability assignment $m_1 \oplus m_2$, describing the combined effect of m_1 and m_2. The basic principle of this rule is demonstrated in Figure 7; such a figure is called an intersection tableau. In front of each row of the intersection tableau a subset of the frame of discernment and the basic probability number assigned to it by the basic probability assignment m_1 are specified. The figure shows only those subsets having a basic probability number not equal to zero. Above the columns of the intersection tableau all subsets of Θ are specified again, but this time with their basic probability numbers according to m_2. The crossing of a row and a column now contains the intersection of the sets associated with the row and

column concerned, and specifies the product of the two basic probability numbers associated with these sets. So, at the crossing of the row corresponding with the set {*mūs'tahab, mūbāh* } having the basic probability number

0.2, and the column corresponding with the set { *mūs'tahab, makrooh* } with the basic probability number 0.9, we find the set. { *mūs'tahab* } with the number 0.18.

m_2 / m_1	***{mūs'tahab,makrooh}* (0.9)**	**Θ (0.1)**
***{mūs'tahab, mūbāh }* (0.2)**	*{ mūs'tahab }* (0.18)	*{ mūs'tahab, mūbāh }* (0.02)
Θ (0.8)	*{mūs'tahab, makrooh }* (0.72)	Θ (0.08)

Figure7: Intersection tableau for m_1 and m_2.

Now observe that the set {*mūs'tahab*}is also present at other places in the tableau since there are various possibilities for choosing two sets X, Y$\subseteq$ Θ such that X $\cap$ Y = {*mūs'tahab* }. Dempster's rule of combination now sums all basic probability numbers assigned to the set {*mūs'tahab*}. The result of this computation is the basic probability number assigned by the orthogonal sum $m_1 \oplus m_2$ to that specific set. The intersection tableau in Figure 7 shows all sets having a probability number not equal to zero. So we have obtained the following probability assignment:

$$m_1 \oplus m_2(X)= \begin{cases} 0.08 \text{ if } X = \Theta \\ 0.18 \text{ if } X = \{\text{mūs'tahab}\} \\ 0.02 \text{ if } X = \{\text{mūs'tahab}, \text{mūbāh}\} \\ 0.72 \text{ if } X = \{\text{mūs'tahab, makrooh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

However, in computing the combination of the two basic probability assignments, as demonstrated above, we may encounter a problem.

Consider m_1 once more and the basic probability assignment m_3 defined by

$$m_3(x) = \begin{cases} 0.4 \text{ if } X = \Theta \\ 0.6 \text{ if } X = \{\text{makrooh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

m_3 / **m_1**	***{ makrooh }* (0.6)**	**Θ (0.4)**
***{mūs'tahab, mūbāh }* (0.2)**	$\varnothing$ (0.12)	*{ mūs'tahab, mūbāh }* (0.08)
Θ (0.8)	*{ makrooh }* (0.48)	Θ (0.32)

Figure 8: An erroneous intersection tableau for m_1 and m_3.

The above figure (figure 8) shows an intersection tableau which has been constructed using the same procedure as before. However, in this erroneous intersection tableau a basic probability assignment greater than zero has been assigned to the empty set; we have that $m_1 \oplus m_3(\varnothing) = 0.12$. So the function $m_1 \oplus m_3$ is not a basic probability assignment, since it does not satisfy the axiom $m_1 \oplus m_3(\varnothing) = 0$. Dempster's rule of combination now simply sets $m_1 \oplus m_3(\varnothing) = 0$. As a consequence, the second axiom is violated; we now have that

$$\sum_{X \subseteq \Theta} m_1 \oplus m_3(X)$$

is less than instead of equal to 1. To remedy this problem, Dempster's rule of combination divides the remaining numbers by the normalization coefficient

$$1- \sum_{Y \cap Z = \varnothing} m1(Y)\, m2(Z)$$

in this example the value of normalization coefficient is 1-0.12 = 0.88. The correct intersection tableau for m_1 and m_3 is depicted in figure 9.

m_3 / m_1	***{ makrooh }* (0.6)**	**Θ (0.4)**
***{mūs'tahab, mūbāh }* (0.2)**	$\varnothing$ (0)	*{ mūs'tahab, mūbāh }* (0.09)
Θ (0.8)	*{ makrooh }* (0.55)	Θ (0.36)

Figure 9: The correct intersection tableau for m_1 and m_3.

From figure 9, we have obtained the following probability assignment:

$$m_1 \oplus m_3\,(X) = \begin{cases} 0.36 \text{ if } X = \Theta \\ 0.55 \text{ if } X = \{\text{ makrooh }\} \\ 0.09 \text{ if } X = \{\text{ mūs'tahab}, \text{mūbāh }\} \\ 0 \quad \text{otherwise} \end{cases}$$

m_3 \ m_2	{ makrooh } (0.6)	Θ (0.4)
{mūs'tahab,makrooh} (0.9)	{ makrooh } (0.54)	{mūs'tahab,makrooh} (0.36)
Θ (0.1)	{makrooh} (0.06)	Θ (0.04)

Figure 10: Intersection tableau for m_2 and m_3

From figure 10, we have obtained the following probability assignment:

$$m_2 \oplus m_3(X) = \begin{cases} 0.04 \text{ if } X = \Theta \\ 0.6 \text{ if } X = \{\text{makrooh}\} \\ 0.36 \text{ if } X = \{\text{mūs'tahab, makrooh}\} \\ 0 \quad \text{otherwise} \end{cases}$$

8. Belief function and belief interval

For a given basic probability assignment, we now define a new function, describing the cumulative belief in a set of hypotheses.

Let Θ be a frame of discernment, and let m be a basic probability assignment on Θ. Then the *belief function* (or *credibility function*) corresponding with m is the function Bel: $2^\Theta \rightarrow [0,1]$ defined by

$$\text{Bel}(X) = \sum_{Y \subseteq X} m(Y)$$

for each $X \subseteq \Theta$.

Some properties of the belief function are:

(1) $Bel(\phi) = 0$
(2) $Bel(\Theta) = 1$
(3) $0 \leq Bel(X) + Bel(\sim X) \leq 1$, where X is a hypothesis and ~X is its negation.

The combination of two belief functions Bel_1 and Bel_2 i.e., $Bel_1 \oplus Bel_2$ is the function $Bel_1 \oplus Bel_2: 2^{\Theta} \rightarrow [0,1]$ defined by

$$Bel_1 \oplus Bel_2(X) = \sum_{Y \subseteq X} m_1 \oplus m_2(Y)$$

A belief function provides for each set X only a lower bound to the 'actual' belief in X. It is also possible that belief has been assigned to a set Y such that $X \subseteq Y$. Therefore, in addition to the belief function the Dempster-Shafer theory defines another function corresponding with a basic probability assignment.

Definition: Let Θ be a frame of discernment and let m be a basic probability assignment on Θ. Then the plausibility function or upper probability function corresponding to m is the function $Pl: 2^{\Theta} \rightarrow [0,1]$ defined by

$$Pl(X) = \sum_{X \cap Y \neq \varnothing} m(Y)$$

for each $X \subseteq \Theta$.
Some properties of the plausibility function are:

(1) $Pl(\phi) = 0$
(2) $Pl(\Theta) = 1$
(3) $1 \leq Pl(X) + Pl(\sim X) \leq 2$

(4) $Pl(X) \geq Bel(X)$

The combination of two plausibility functions Pl_1 and Pl_2 i.e., $Pl_1 \oplus Pl_2$ is the function $Pl_1 \oplus Pl_2: 2^{\Theta} \rightarrow [0,1]$ defined by

$$Pl_1 \oplus Pl_2(X) = \sum_{X \cap Y \neq \varnothing} m_1 \oplus m_2(Y)$$

The function value Pl(X) indicates the total confidence not assigned to ~X, so Pl(X) provides an upper bound to the 'real' confidence in X. For a given basic probability assignment m, the property

$Pl(X) = 1 - Bel(\sim X)$

for each $X \subseteq \Theta$, holds for the belief function Bel and the plausibility function Pl corresponding to m. The difference Pl(X) - Bel(X) indicates the confidence in the sets Y for which $X \subseteq Y$ and therefore expresses the ignorance about X.

Definition: Let Θ be a frame of discernment and let m be a basic probability assignment on Θ. Let Bel be the belief function corresponding to m, and let Pl be the plausibility function corresponding to m. For each $X \subseteq \Theta$, the closed interval [Bel(X), Pl(X)] is called the belief interval of X

9. Interpretation of Belief Interval

Belief interval can be interpreted in terms of belief in a hypothesis, belief in negation of the hypothesis, plausibility of the hypothesis and ignorance about the hypothesis.

Some of belief intervals with their interpretations are given below:

Let Θ be a frame of discernment, and let $X \subseteq \Theta$. Now,

(1) If [Bel(X), Pl (X)] = [0,0], then no information concerning X is available while information concerning ~X is sufficient to lead the belief in ~X to 100%, i.e.,
   (a) there is no evidence for X and therefore Bel(X) = 0
   (b) since the value of Pl(X) = 0, therefore Bel(~X) = 1 – Pl(X) = 1 or 100% which shows that there is sufficient evidence against X
   (c) Pl(X) – Bel(X) = 0, showing zero % ignorance about X (or 100% awareness)
(2) If [Bel(X),Pl (X)] = [0,1], then no information concerning X or against X is available, i.e.,
   (a) there is no evidence for X and therefore Bel(X) = 0
   (b) since the value of Pl(X) = 1, therefore Bel(~X) = 1 – Pl(X) = 0 which shows that there is no evidence against X
   (c) Pl(X) – Bel(X) = 1 – 0 = 1, showing 100 % ignorance about X
(3) If [Bel(X), Pl(X)] = [1,1], then X has been completely confirmed, i.e.,
   (a) Bel(X) = 1 which indicates that there is sufficient evidence to lead our belief in X to 100%
   (b) since the value of Pl(X) = 1, therefore Bel(~X) = 1 – Pl(X) = 0 which shows that there is no evidence against X
   (c) Pl(X) – Bel(X) = 1 – 1 = 0, showing zero% ignorance about X

(4) If [Bel(X), Pl(X)] = [0.5,0.5], then 50% information concerning X and 50% against X is available, i.e.,
   (a) there is 50% evidence in favour of X and therefore Bel(X) = 0.5
   (b) since the value of Pl(X) = 0.5, therefore Bel(~X) = 1 – Pl(X) = 0.5 which shows that there is also an equal amount of evidence, i.e. 50% against X
   (c) Pl(X) – Bel(X) = 0.5 – 0.5 = 0, showing zero% ignorance about X

(5) If [Bel (X), Pl (X)] = [0.2,1], then there is some evidence in favour of the hypothesis X is available while no information is available against X, i.e.,
   (a) there is such an amount of evidence for X, that causes the belief in X or Bel(X) = 0.2
   (b) since the value of Pl(X) = 1, therefore Bel(~X) = 1 – 1 = 0 which shows that there is no evidence against X
   (c) Pl(X) – Bel(X) = 1 – 0.2 = 0.8, showing 80% ignorance about X

(6) If [Bel (X), Pl (X)] = [0.3,0.8], then we have some evidence in favour of X as well as some against X, i.e.,
   (a) there is such an amount of evidence for X, that causes the belief in X or Bel(X) = 0.3
   (b) since the value of Pl(X) = 0.8, therefore Bel(~X) = 1 – 0.8 = 0.2 which shows that there 20% evidence against X
   (c) Pl(X) – Bel(X) = 0.8 – 0.3 = 0.5, showing 50% ignorance about X

10. Belief in Islamic Perspective

In section 7, there are different values of basic probability assignments from which one can determine different values of belief and plausibility. The values of belief and plausibility, concerning different focal elements in m_1, m_2, m_3, $m_1 \oplus m_2$, $m_2 \oplus m_3$ and $m_1 \oplus m_3$ are provided below:

Bel_1(*{ mūs'tahab, mūbāh }*) = 0.2
Pl_1(*{ mūs'tahab, mūbāh }*) =0.2+0.8 = 1

Bel_2(*{mūs'tahab, makrooh }*) = 0.9
Pl_2(*{mūs'tahab, makrooh }*) = 0.9 + 0.1 = 1

Bel_3(*{ makrooh }*) = 0.6
Pl_3(*{makrooh }*) = 0.6 + 0.4 = 1

$Bel_1 \oplus Bel_2$(*{ mūs'tahab }*) = 0.18
$Pl_1 \oplus Pl_2$(*{ mūs'tahab }*) = 0.18 + 0.02 + 0.72 + 0.08 = 1

$Bel_1 \oplus Bel_2$(*{ mūs'tahab, mūbāh }*) = 0.18 + 0.02 = 0.2
$Pl_1 \oplus Pl_2$(*{ mūs'tahab, mūbāh }*) = 0.18 + 0.02 + 0.72 + 0.08 = 1

$Bel_1 \oplus Bel_2$(*{mūs'tahab, makrooh }*) = 0.72 + 0.18 = 0.9
$Pl_1 \oplus Pl_2$(*{mūs'tahab, makrooh }*) = 0.18 + 0.02 + 0.72 + 0.08 = 1

$Bel_2 \oplus Bel_3$(*{ makrooh }*) = 0.6
$Pl_2 \oplus Pl_3$(*{ makrooh }*) = 0.6 + 0.36 + 0.04 = 1

$Bel_2 \oplus Bel_3$(*{mūs'tahab, makrooh }*) = 0.36 + 0.6 = 0.96
$Pl_2 \oplus Pl_3$(*{mūs'tahab, makrooh }*) = 0.36 + 0.6 + 0.04 = 1

$Bel_1 \oplus Bel_3$(*{ makrooh }*) = 0.55
$Pl_1 \oplus Pl_3$(*{ makrooh }*) = 0.55 + 0.36 = 0.91

$Bel_1 \oplus Bel_3$(*{ mūs'tahab, mūbāh }*) = 0.09
$Pl_1 \oplus Pl_3$(*{ mūs'tahab, mūbāh }*) = 0.09 + 0.36 = 0.45

Finally, above values lead to form different belief intervals e.g., the belief interval for
{ mūs'tahab } is:
[$Bel_1 \oplus Bel_2$ (*{ mūs'tahab }*), $Pl_1 \oplus Pl_2$(*{ mūs'tahab }*)] = [0.18,1]
and the belief interval for *{makrooh }* is:
[$Bel_2 \oplus Bel_3$ (*{makrooh }*), $Pl_2 \oplus Pl_3$ (*{makrooh }*)] = [0.6,1]
From the two intervals, it is observed that:
Bel(*{ makrooh }*) > Bel(*{ mūs'tahab }*)

The belief interval of *{ mūs'tahab }* indicates that:

(a) there is such an amount of evidence in favour of *{ mūs'tahab }* that causes the belief in *{ mūs'tahab }* or Bel (*{ mūs'tahab }*) = 0.18
(b) since the value of Pl(*{ mūs'tahab }*) = 1, therefore Bel(Not (*{ mūs'tahab }*)) = 1 – Pl(*{ mūs'tahab }*) = 0 which shows that there is no evidence against *{ mūs'tahab }*
(c) Pl(X) – Bel(X) = 1 – 0.18 = 0.82, showing 82% ignorance about *{ mūs'tahab }*

Similarly, the belief interval of *{makrooh }* indicates that:

(a) there is such an amount of evidence in favour of *{makrooh }* that causes the belief in *{makrooh }* or Bel (*{ makrooh }*) = 0.6

(b) since the value of Pl(*{makrooh }*)= 1, therefore Bel(Not (*{ makrooh }*)) = 1– Pl(*{ makrooh }*) = 0 which shows that there is no evidence against *{makrooh }*

(c) Pl(X) – Bel(X) = 1 – 0.6 = 0.4, showing 40% ignorance about *{ makrooh }*

These belief intervals provide different alternatives of belief that are helpful while making a decision about how to act on a belief.

Now, degree of belief is a function,

$$\text{degree of belief: strength of evidence} \rightarrow [0,1]$$

The strength of evidence is the product of sufficient quantity of evidence and quality of evidence; that is,

$$\text{strength of evidence} = \text{sufficient quantity of evidence} \times \text{quality of evidence.}$$

It lies in the closed interval [0,1].

The sufficient quantity of evidence and the quality of evidence are fuzzy and therefore a degree of membership is required for each to define.

Any increase in strength of evidence causes an increase in degree of belief. The proportion, or fraction, of any increase in strength of evidence, which is believed is called the marginal propensity to believe (MPB) Or, alternatively stated, the MPB is the ratio of an increase in degree of belief to the increase in strength of evidence which brought the strength of belief increase about; that is,

MPB = increase in degree of belief / increase in strength of evidence
or,

$$MPB = d(\text{degree of belief})/d(\text{strength of evidence})$$

It represents the slope of the linear curve (figure 11 or 12) and typically, it is expected to be unity for a rational person. Different persons behave differently with the strength of evidence. Some believe a little for too much strength of evidence while others believe too much for a little strength of evidence.

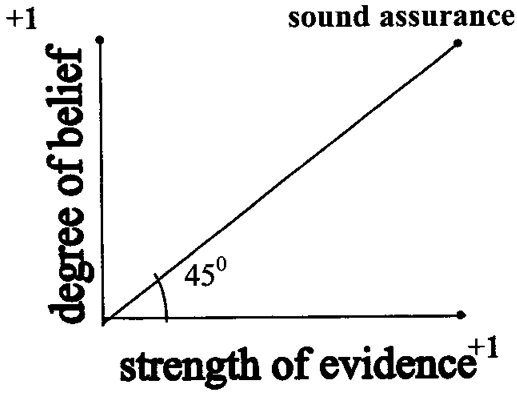

Figure 11: A linear relationship between degree of belief and strength of evidence that leads to sound assurance. The relationship is for authentic evidence and it shows a rational behaviour throughout (MPB=1 at each point)

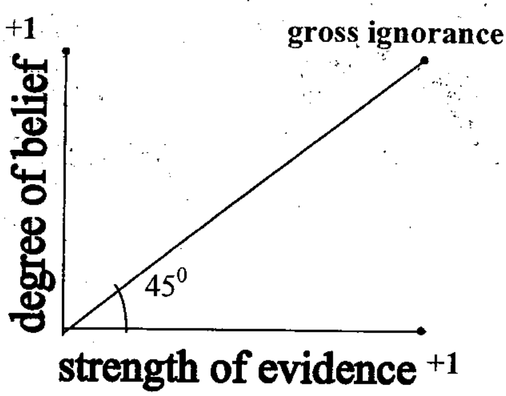

Figure 12: A linear relationship between degree of belief and strength of evidence that leads to gross ignorance. The relationship is for unauthentic evidence, although it shows a rational behaviour

(**Note:** for non-linear relationships, there are so many paths to reach the sound assurance or gross ignorance)

The behaviour of a person at a point is either rational or irrational. The behaviour is rational if MPB =1 at that point while for MPB ≠ 1, his behaviour is either credulity (*sūr'at-ūl-e'tiqādī*) or incredulity (*būtoo'-ūl-e'tiqādī*). A person with MPB <1,on some interval, is incredulous (*batī'-ūl-e'tiqād*) at that interval i.e. he believes a little for too much strength of evidence in the interval while a person with MPB >1, on some interval, is credulous (*sarī'-ūl-e'tiqād*) at that interval i.e. he believes too much for a little strength of evidence in the interval.

Although belief depends on evidence but evidence is also needed to believe, i.e. belief and evidence are implicit functions. This is so, because evidence may be either authentic or unauthentic. If certain evidence supports a truth or is against an untruth then it is authentic. On the other hand, if it supports an untruth or is against a truth, then it is unauthentic. Authentic evidence, with 100% strength, leads to sound assurance, provided the confidence in the evidence is 100% and is invariant.

Unauthentic evidence with 100% strength leads to gross ignorance provided the confidence in the evidence is 100% and is invariant.

Authentic or unauthentic evidence with 100% strength leads to conformity provided the confidence in the evidence is 100% but variant.

Since belief depends on evidence, therefore the belief interval

[Bel(H), Pl(H)] (see figure 13)

may vary when some more evidence is received, that causes an increase in Bel(H). But the interval may also vary in the sense that Bel(H) may reduce. This is so because if some of the evidences in favour of H, among those that are used to form the given interval, is unauthentic and when someone comes to know the fact about such evidences, his belief in H reduces and therefore in consequence, the ignorance, Pl(H)-Bel(H), reduces.

Similarly, if some of the evidences against H, among those that are used to form the given interval, is unauthentic and when he comes to know the fact about such evidences, his belief in ~H reduces and therefore in consequence, the plausibility, 1-Bel(~H), reduces.

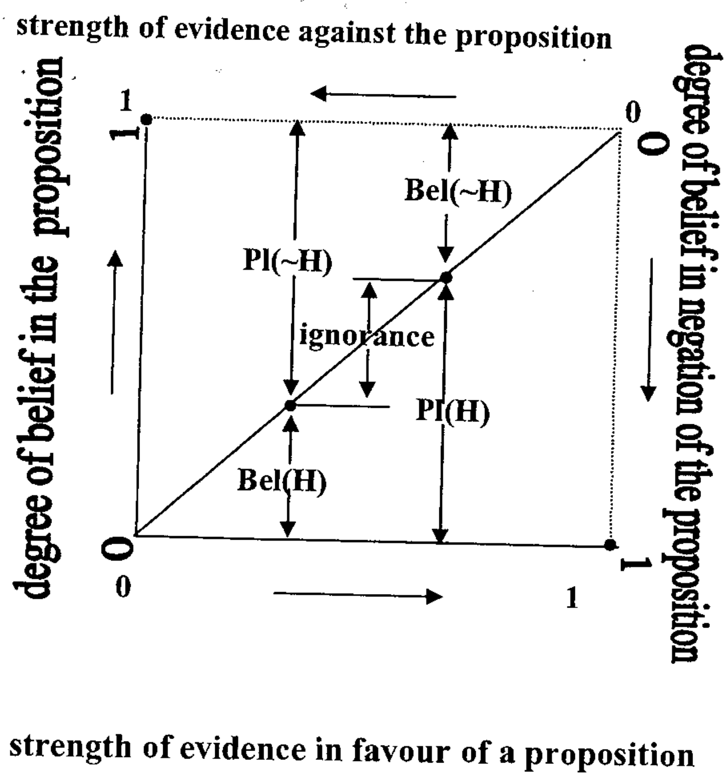

Figure 13: A linear relationship between 'strength of evidence' and 'belief together with plausibility and ignorance'

Mūhaddisīn dealt 'strength of evidence' very carefully while confirming the authenticity of a narration. This was so as they were very cautious about 'quality of evidence' (*sahīh, hasan, za'īf, mau'zoo'* etc.) as well as about 'quantity of evidence' (*mūtawātar, mashhoor, 'azīz, gharīb* etc.)

Their belief interval is given by:
[Bel(H),Pl(H)] = [a,1], where $0.5 < a \leq 1$ and Bel(~H) = 0 (see figure 14)

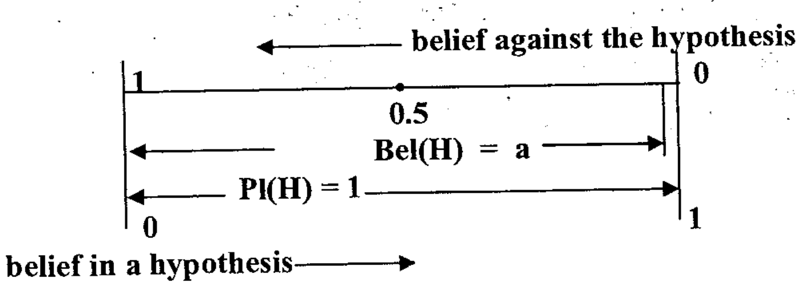

Figure 14: belief interval for mūhaddisīn

Their behaviour was rational i.e., they were neither credulous nor incredulous. Their degree of belief 'a', in the authenticity of a narration was greater than 50% and was closed to 1 in most of the cases. Their degree of belief against the authenticity of the narration was almost zero and their ignorance '1-a' was also very closed to zero. Moreover, as they acquired authentic evidence, so their degree of belief approached to sound assurance.

11. Decision based on Evidence

In the previous section, the use of Dempster-Shafer theory of evidence and the Dempster's rule of combination enable us at the first step to get different alternatives of beliefs, after receiving some evidence. These approaches allow the narrowing and revision of such beliefs in the light of more evidence, but suppose after reaching at the current situation, our goal is now only to decide what to accept among the two alternatives *{mūs'tahab}* and *{makrooh}*.

Although, our initial goal was to investigate whether the deed is *mūbāh, mūs'tahab,* or *makrooh,* which can be achieved after receiving some more evidence. The sets *{mūs'tahab}* and *{makrooh}* are the only two among the different alternatives, which are singleton. Since from the previous section, it can be observed that:

(1) Bel*({ makrooh })* > Bel*({ mūs'tahab })* and,
(2) Bel*({not makrooh })* = Bel*({ not mūs'tahab })* = 0.

Moreover, Bel*(makrooh) > 50%,* therefore it can be concluded that the deed is more likely to be *makrooh*
In general, if there is:

(1) insufficient evidence for both X and Y, i.e., $0 \leq \text{Bel}(X) < 0.5$ and $0 \leq \text{Bel}(Y) < 0.5$,
then neither X will be accepted nor Y.

(2) sufficient evidence for either X or Y, i.e., either
   (i) $0 \leq \text{Bel}(X) \leq 0.5$ and $0.5 < \text{Bel}(Y) \leq 1$, or
   (ii) $0 \leq \text{Bel}(Y) \leq 0.5$ and $0.5 < \text{Bel}(X) \leq 1$,
then the proposition having sufficient evidence will be accepted.

(3) 50 % evidence for both X and Y, i.e., $\text{Bel}(X) = \text{Bel}(Y) = 0.5$ and evidence against X and Y are different, then the hypothesis, against which the evidence is lesser will be accepted.

(4) 50 % evidence for both X and Y, i.e., $\text{Bel}(X) = \text{Bel}(Y) = 0.5$ and evidence against both are same, i.e., $\text{Bel}(\sim X) = \text{Bel}(\sim Y)$ and if

(4.1) one of the hypotheses is makrooh, then the hypothesis makrooh will be accepted.

(4.2) one of the hypotheses is mūbāh and the other one is mūs´tahab, then the hypothesis mūs´tahab will be accepted.

Glossary

akhlāq: ethics
a'māl: plural of *amal*
amal: conduct
'aqā'id: plural of *'aqīdah*
'aqīdah: faith
batī '-ūl-e'tiqād: incredulous
būtoo'-ūl-e'tiqādī: incredulity
dalīl-e-qat'ī: definitive evidence
dalīl-e-zannī: conjectural or presumptive evidence
farḍ: obligatory
ghair-harām: lawful
ghair-ikh'tīārī: out of control
ghair-muttafaq-un- alaih: antonym of *muttafaq-un-alaih*
harām: unlawful
'ibādāt: *plural of 'ibādat*
'ibādat: worship
ikh'tīārī: under control
jehl-e-mūrak'kab: gross ignorance
makrooh-e-tahrīmī: unallowable
makrooh-e-tanzīhī: abominable
mū'āmalāt: plural of *mū'āmalah*
mū'āmalah: deal
mū'āsharat: way of living
mūbāh: permissible
mūs'tahab: desirable
sarī '-ūl-e'tiqād: credulous
shak: doubt
sūr'at-ūl-e'tiqādī: credulity
tajassūs: curiousity
takhay'yūl: imagination
taqlīd: conformity
wahm: hallucination

wājib: unavoidable
yaqīn-e-kāzib: see ***jehl-e-murak'kab***
yaqīn-e-sādiq: sound assurance
zann: presumption

References

1- Rich, Elaine and Knight, Kevin (1991). *Artificial Intelligence,* New York: McGraw-Hill, Inc.
2- Tanimoto, Steven L. (1990). *The Elements of Artificial Intelligence,* New York: Computer Science Press (An imprint of W.H. Freeman and Company)
3- Luger, George F. and Stubblefield, William A. (1998). *Artificial Intelligence,* England: Addison Wesley Longman, Inc.
4- Sharan, S. N. (1993). *Fundamentals of Expert Systems,* Delhi: CBS Publishers and Distributors
5- Turban, Efraim (1995). *Decision Support and Expert Systems,* New Jersey: Prentice Hall
6- Lucas, Peter and Van Der Gaag, Linda (1991). *Fundamentals of Expert Systems,* England: Addison-Wesley Publishing Company, Inc.
7- Ayub, Allama Hafiz Muhammad. *Fitna-e-inkār-e-hadīs,* Karachi: Maktaba-e-Rāzi
8- Porkess, Roger (1993).*Dictionary of Statistics,* London: Collins
9- Walpole, Ronald E. (1990). *Introduction to Statistics,* New York: Macmillan Publishing Co., Inc.
10- Chaudhry, Sher Muhammad (1984). *Introduction to Statistical Theory,* Lahore: Ilmi Kitāb Khāna
11- Dougherty, Edward R. *Mathematical Methods for Artificial Intelligence and Autonomous Systems,* New York: Prentice-Hall International, Inc.
12- McConnel, Campbell R. (1975). *Economics,* New York: McGraw-Hill book company

13-Klir, George J., Yuan, Bo (1997). *Fuzzy Sets and Fuzzy Logic, Theory and Applications,* New Delhi-110001: Prentice Hall of India (Pvt. Ltd.)